

Illustration - 15 A 250 cm^3 glass bottle is completely filled with water at 50°C . The bottle and water are heated to 60°C . How much water runs over if :

- (a) the expansion of the bottle is neglected?
 (b) the expansion of the bottle is included? Given the coefficient of areal expansion of glass

$$\gamma_g = 1.2 \times 10^{-5} / ^\circ\text{C} \text{ and } \gamma_{\text{water}} = 60 \times 10^{-5} / ^\circ\text{C}.$$

SOLUTION :

Water overflow = (final volume of water)
 – (final volume of bottle)

- (a) If the expansion of bottle is neglected :

$$\begin{aligned} \text{Water overflow} &= 250 (1 + \gamma_\ell \theta) - 250 \\ &= 250 \times 60 \times 10^{-5} \times 10 \\ \Rightarrow \text{water overflow} &= 1.5 \text{ cm}^3. \end{aligned}$$

- (b) If the bottle (glass) expands :

$$\begin{aligned} \text{Water overflow} &= (\text{final volume of water}) \\ &\quad - (\text{final volume of glass}) \\ &= 250 (1 + \gamma_\ell \theta) - 250 (1 + \gamma_g \theta) = 250 (\gamma_\ell - \gamma_g) \theta \\ &= 250 (58.8 \times 10^{-5}) \times (60 - 50) \\ \Rightarrow \text{water overflow} &= 1.47 \text{ cm}^3. \end{aligned}$$

TRANSMISSION OF HEAT

Section - 3

There are three different ways in which heat can be transferred: conduction, convection and radiation.

Conduction :

It is a process by which heat is transferred in solids. In conduction, molecules vibrate about a fixed location and transfer the heat by collision.

Convection :

It is a process by which heat is transferred in fluids (liquids & gases). In convection, transfer of heat takes place by transport of matter (in form of motion of particles).

When a liquid in a container is heated, the molecules at the lower layers are heated up and the density decreases. As a result the molecules rise up and heavier ones come down and hence a continuous movement of molecules takes place giving rise to convection currents. In this manner the whole of liquid gets heated.

Radiation :

It is a process in which the heat is transferred in the form of electromagnetic waves without the aid of any material medium. Heat is radiated from a body due to its own temperature (*greater than 0 K*) is called as **radiant energy**.

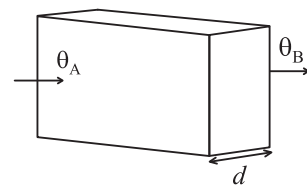
Thermal Conductivity

In solids, heat is transferred through conduction. We will study conduction of heat through a solid bar in the following section.

Consider a solid bar of thickness d and area of cross-section A . The left side of the bar is maintained at a temperature θ_A and right side at θ_B .

Let us assume that $\theta_A > \theta_B$.

Heat flows from high temperature to low temperature i.e. *from left side to right side, as shown.*



After some time, temperature of each section becomes constant with time. This is known as *steady state*.

If ΔQ is the amount of heat flow (transferred) through a cross-section in Δt seconds at steady state, then the rate of heat flow is given by :

$$H = \frac{\Delta Q}{\Delta t} = \frac{KA(\theta_A - \theta_B)}{d}$$

K = coefficient of thermal conductivity of solid material

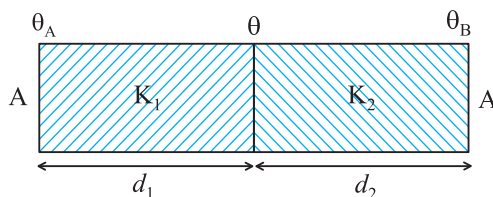
Its units are : $\text{cal s}^{-1} \text{m}^{-1} \text{K}^{-1}$ or $\text{J s}^{-1} \text{m}^{-1} \text{K}^{-1}$ ($\text{W m}^{-1} \text{K}^{-1}$)

Heat Conduction through a Composite Slab (rod)

(A) Two rods connected in series

(B) Three rods connected in series

(A) Consider a composite rod made up of two rods of lengths d_1 and d_2 and each of cross-section A , joined end to end (as shown). Let K_1 and K_2 be the coefficients of thermal conductivities of two rods and θ_A and θ_B be the temperatures of two ends of the composite rod.



Let us assume θ as the temperature of the junction of two rods and $\theta_A > \theta_B$. Heat flows from left to the right. In steady state, heat flow per second is same through each rod. It is given by :

$$H = \frac{\Delta Q}{\Delta t}$$

For Ist rod : $H = \frac{K_1}{d_1} A (\theta_A - \theta)$ (i)

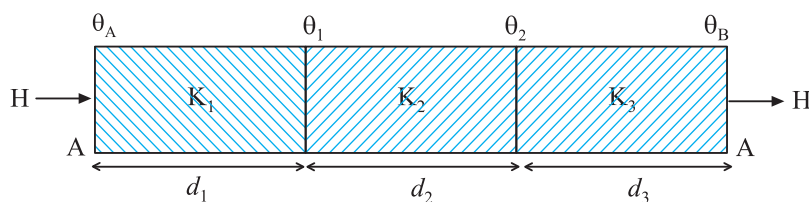
For IInd rod : $H = \frac{K_2}{d_2} A (\theta - \theta_B)$ (ii)

Eliminating θ from (i) and (ii) we get : $H = \frac{A(\theta_A - \theta_B)}{\frac{d_1}{K_1} + \frac{d_2}{K_2}}$

Equating (i) and (ii) we get :

$$\frac{K_1}{d_1} A (\theta_A - \theta) = \frac{K_2}{d_2} A (\theta - \theta_B) \Rightarrow \theta = \frac{\frac{K_1}{d_1} \theta_A + \frac{K_2}{d_2} \theta_B}{\frac{K_1}{d_1} + \frac{K_2}{d_2}}$$

- (B) Consider a composite slab made up of three rods of lengths d_1 , d_2 and d_3 joined end to end. Let area of cross-section of each rod be A . The left side of the composite rod is maintained at θ_A and right side at θ_B ($\theta_A > \theta_B$). Let the coefficient of thermal conductivities of three rods be K_1 , K_2 and K_3 and the junction temperatures of rods 1 and 2 be θ_1 and that of 2 and 3 be θ_2 .



As $\theta_A > \theta_B$ heat flows from left to right side. At steady state let H be the heat flowing per second.

$$\text{First rod : } H = \frac{K_1}{d_1} A (\theta_A - \theta_1) \quad \dots\dots\dots(i)$$

$$\text{Second rod : } H = \frac{K_2}{d_2} A (\theta_1 - \theta_2) \quad \dots\dots\dots(ii)$$

$$\text{Third rod : } H = \frac{K_3}{d_3} A (\theta_2 - \theta_B) \quad \dots\dots\dots(iii)$$

$$\text{From (i) } \theta_A - \theta_1 = \frac{H}{A} \frac{d_1}{K_1} \quad \dots\dots\dots(iv)$$

$$\Rightarrow \theta_1 = \theta_A - \frac{H}{A} \frac{d_1}{K_1}$$

$$\text{From (ii) } \theta_1 - \theta_2 = \frac{H}{A} \frac{d_2}{K_2} \quad \dots\dots\dots(v)$$

$$\text{From (iii) } \theta_2 - \theta_B = \frac{H}{A} \frac{d_3}{K_3} \quad \dots\dots\dots(vi)$$

$$\Rightarrow \theta_2 = \theta_B + \frac{H}{A} \frac{d_3}{K_3}$$

θ_A and θ_B are known, so θ_1 and θ_2 can be determined.

Adding the equations (iv), (v) and (vi) to eliminate θ_1 and θ_2 , we get :

$$\theta_A - \theta_B = \frac{H}{A} \left(\frac{d_1}{k_1} + \frac{d_2}{k_2} + \frac{d_3}{k_3} \right)$$

$$\Rightarrow H = \frac{(\theta_A - \theta_B) A}{\frac{d_1}{k_1} + \frac{d_2}{k_2} + \frac{d_3}{k_3}}$$

Conduction of Heat through a Cylindrical Shell

θ_1 = temperature of the inner surface

θ_2 = temperature of the outer surface

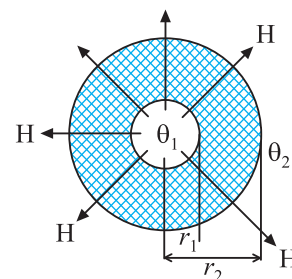
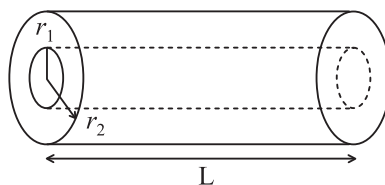
L = length of the cylinder

K = thermal conductivity

r_1 and r_2 are inner and outer radii of the cylinder

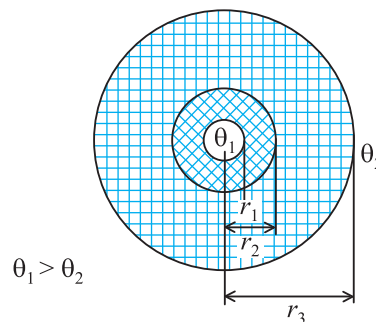
$$\frac{\Delta Q}{\Delta t} = \text{Heat flow per second} = H$$

$$H = \frac{2\pi L (\theta_1 - \theta_2)}{\frac{1}{K} \log \frac{r_2}{r_1}}$$



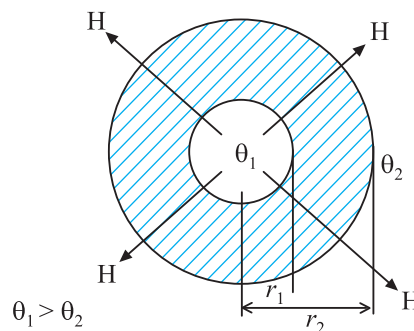
In case of two cylindrical layers

$$H = \frac{2\pi L (\theta_1 - \theta_2)}{\frac{1}{K_1} \log \frac{r_2}{r_1} + \frac{1}{K_2} \log \frac{r_3}{r_2}}$$



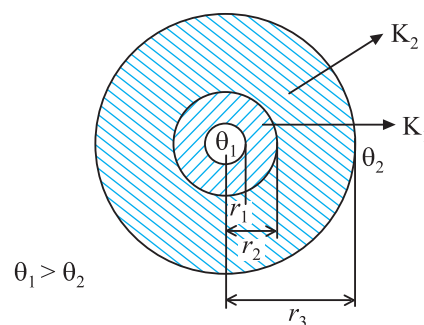
Heat Conduction through a Spherical Shell

$$H = \frac{4\pi (\theta_1 - \theta_2)}{\frac{1}{K} \left(\frac{1}{r_1} - \frac{1}{r_2} \right)}$$



In case of multiple shells

$$H = \frac{4\pi (\theta_1 - \theta_2)}{\frac{1}{K_1} \left(\frac{1}{r_1} - \frac{1}{r_2} \right) + \frac{1}{K_2} \left(\frac{1}{r_2} - \frac{1}{r_3} \right)}$$



Growth of Ice on the Surface of a Lake

A layer of ice of thickness x is formed on the surface of a lake. The water below the ice layer is at a temperature of 0°C . The air above the ice layer is at a temperature of $-\theta^\circ\text{C}$ (below 0°C). Heat flows from water to air through ice. As a result of this heat loss, the layer increase in thickness from below.

Let dx be the increase in thickness in time dt .

$$\begin{aligned}\Rightarrow \text{heat loss from water to air} &= (\text{mass}) \times (\text{latent heat}) \\ &= (A dx) \rho L \\ \text{where } \rho &: \text{density of ice ;}\end{aligned}$$

L : latent heat of ice ;

A : area of cross-section of ice slab

Using $\frac{\Delta Q}{\Delta t} = \frac{KA(\theta_1 - \theta_2)}{d}$

$$\Rightarrow \text{Heat flow per second} = A\rho L \left(\frac{dx}{dt} \right) = \frac{KA\theta}{x}$$

$$\text{Rate of growth of thickness} = \frac{dx}{dt} = \frac{K\theta}{\rho L x}$$

If the thickness of ice layer changes from x_1 to x_2 in t seconds, then :

$$\begin{aligned}\int_{x_1}^{x_2} x dx &= \frac{K\theta}{\rho L} \int_0^t dt \\ \Rightarrow \frac{x_2^2 - x_1^2}{2} &= \frac{K\theta t}{\rho L}\end{aligned}$$

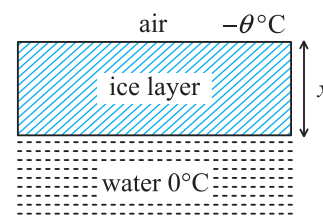


Illustration - 16 Water is being boiled in a flat bottom kettle placed on a stove. The area of the bottom is 3000 cm^2 and the thickness is 2 mm . If the amount of steam produced is 1 g/min , calculate the difference of temperature between the inner and outer surface of the bottom. K for the material of kettle is $0.5 \text{ cal/}^\circ\text{C/s/cm}$, and the latent heat of steam is 540 cal/gm .

SOLUTION :

$$\begin{aligned}\text{Mass of steam produced} &= \frac{dm}{dt} = \frac{1}{60} \text{ g/s} & \Rightarrow \frac{dQ}{dt} &= \frac{KA\theta}{d} \\ \text{Heat transferred per sec.} &= \frac{dQ}{dt} = L \frac{dm}{dt} & \Rightarrow L \frac{dm}{dt} &= \frac{KA\theta}{d} \\ \Rightarrow \frac{dQ}{dt} &= 540 \times \frac{1}{60} \text{ cal/s} = 9 \text{ cal/s} & \Rightarrow 9 &= \frac{0.5 \times 3000 \times \theta}{0.2} \\ \text{Area} &= 3000 \text{ cm}^2 ; \quad K = 0.5 \text{ cal/}^\circ\text{C/s/cm} & \Rightarrow \theta &= 1.2 \times 10^{-3} ^\circ\text{C} \\ \theta &= \text{temperature difference} \\ d &= \text{thickness} = 2 \text{ mm} = 0.2 \text{ cm}.\end{aligned}$$

Illustration - 17 A uniform copper bar 100 cm long is insulated on sides, and has its ends exposed to the ice and steam respectively. If there is a layer of water 0.1 mm thick at each end, calculate the temperature gradient in the bar. $K_{Cu} = 1.04$ and $K_{water} = 0.0014$ in C.G.S. units.

SOLUTION :

Let θ_1 and θ_2 be the temperatures at the ends of the copper bar.

Heat transfer per sec. through the system is :

$$\frac{dQ}{dt} = \frac{A(100-0)}{\left(\frac{0.01}{K_w} + \frac{100}{K_{Cu}} + \frac{0.01}{K_w}\right)}$$

Heat transfer per sec. through copper bar

$$= \frac{K_{Cu} A(\theta_1 - \theta_2)}{100}$$

As the rods are in series, heat transfer per second must be same through each part.

$$\Rightarrow \frac{A(100-0)}{\left(\frac{0.01}{K_w} + \frac{100}{K_{Cu}} + \frac{0.01}{K_w}\right)} = \frac{K_{Cu} A(\theta_1 - \theta_2)}{100}$$

Putting $K_{Cu} = 1.04$ and $K_w = 0.0014$ we get :

$$\text{Temperature gradient} = \frac{(\theta_1 - \theta_2)}{100} = 0.87^\circ\text{C cm}^{-1}.$$

Illustration - 18 A closed cubical box made of perfectly insulating material has walls of thickness 8 cm and the only way for the heat to enter or leave the box is through the solid, cylindrical, metallic plugs each of cross-sectional area 12 cm^2 and length 8 cm fixed in the opposite walls of the box as shown in fig. The outer surface A is kept at 100°C while the outer surface B of other plug is kept at 4°C . K of the material of the plugs is $0.5\text{ cal/s/}^\circ\text{C/cm}$. A source of energy generating 36 cal/s is enclosed inside the box. Find the equilibrium temperature of the inner surface of the box assuming that it is same at all points on the inner surface.

SOLUTION :

Let θ be the temperature of inner surface of box.

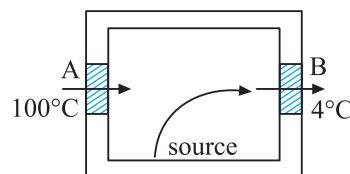
Heat transfer per sec. through A + Heat produced by source per sec.

= Heat transfer per sec. through B

$$\Rightarrow \left(\frac{dQ}{dt}\right)_A + 36\text{ cal/s} = \left(\frac{dQ}{dt}\right)_B$$

$$\Rightarrow \frac{KA(100-\theta)}{d} + 36 = \frac{KA(\theta-4)}{d}$$

$$\Rightarrow KA(\theta-4-100+\theta) = 36 \times d.$$



Now $d = 8\text{ cm}$, $A = 12\text{ cm}^2$, $K = 0.5\text{ cal/s/}^\circ\text{C/cm}$.

$$\Rightarrow 2\theta - 104 = \frac{36 \times 8}{12 \times 0.5}$$

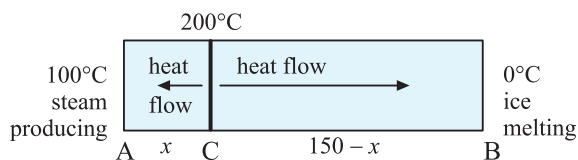
$$\Rightarrow \theta = 76^\circ\text{C}$$

Illustration - 19 One end of a copper rod of uniform cross-section and of length 1.5 m is in contact with ice and the other end with water. At what point along its length should a temperature of 200°C be maintained so that in steady state, the mass of ice melting is equal to that of steam produced in the same interval of time? Assume that whole system is insulated from surroundings.

SOLUTION :

The end A of the copper rod is at 100°C (vapourising water) and end B is at 0°C (melting ice).

Let C be the point where 200°C temperature is maintained ($AC = x$).



$$\text{Heat received per sec at A} = \frac{KA(200-100)}{x}$$

$540 \times \text{mass of steam produced per sec}$

$$= \frac{KA(100)}{x} \quad \dots\dots(i)$$

$$\text{Heat received per sec at B} = \frac{KA(200-0)}{150-x}$$

$80 \times \text{mass of ice melting per sec}$

$$= \frac{KA(200)}{150-x} \quad \dots\dots(ii)$$

Dividing (i) by (ii) we get :

$$\frac{540}{80} = \frac{100}{200} \left(\frac{150-x}{x} \right)$$

$$\Rightarrow \frac{27}{2} = \frac{150-x}{x} \Rightarrow x = 10.34 \text{ cm}$$

Illustration - 20 Two identical rods are connected between two containers one of them is at 100°C and another is at 0°C . If the rods are connected in parallel then the rate of melting of ice is q_1 gm/sec. If they are connected in series then the rate is q_2 . The ratio q_2/q_1 is :

- (A) 2 (B) 4 (C) 1/2 (D) 1/4

SOLUTION : (D)

In parallel, $H = H_1 + H_2$

$$H = q_1 L = \frac{KA(100-0)}{\ell} + \frac{KA}{\ell} (100-0)$$

$$\Rightarrow q_1 L = \frac{2KA}{\ell} (100-0) \quad \dots\dots(i)$$

$$\text{In series, } H = \frac{A(100-0)}{\frac{\ell}{K} + \frac{\ell}{K}} = \frac{KA}{2\ell} (100-0)$$

$$\Rightarrow q_2 \ell = \frac{KA}{2\ell} (100-0) \quad \dots\dots(ii)$$

Divide (ii) by (i) to get :

$$\frac{q_2}{q_1} = \frac{1}{4} \Rightarrow \text{D is correct.}$$

Illustration - 21 A and B are two point on a uniform metal ring whose centre is C and the angle $ACB = \theta$. A and B are maintained at two different constant temperatures. When $\theta = 180^\circ$, the rate of total heat flow from A to B is 1.2 W. When $\theta = 90^\circ$, this rate will be :

- (A) 0.6 W (B) 0.9 W (C) 1.6 W (D) 1.8 W

SOLUTION : (C)

$$H_0 = 2 \frac{KA}{\pi r} (T_A - T_B) = 1.2$$

$$H = H_1 + H_2$$

$$H = \frac{KA}{\pi r/2} (T_A - T_B) + \frac{KA}{3\pi r/2} (T_A - T_B)$$

$$H = \frac{KA}{\pi r} \left(2 + \frac{2}{3} \right) (T_A - T_B)$$

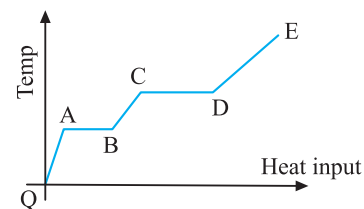
$$H = \frac{H_0}{2} \left(2 + \frac{2}{3} \right) = \frac{4}{3} H_0 = \frac{4}{3} (1.2)$$

$$\Rightarrow H = 1.6 \text{ W. (C is correct).}$$

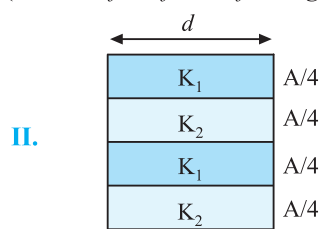
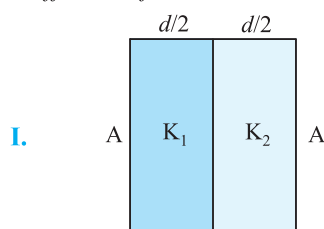
IN-CHAPTER EXERCISE - A

1. A solid material is supplied with heat at a constant rate. The temperature of the material is changing with heat input as shown in the graph. Study the graph carefully and answer the following questions :

- (a) What do the horizontal regions AB and CD represent ?
 (b) If $CD = 2 AB$, what do you infer ?
 (c) What does the slope DE represent ?
 (d) The slope of $OA > \text{slope of } BC$. What does this indicate ?



2. A vacuum pump is connected to an insulating flask, containing 50 gm of water at 0°C . As the water vapour is being pumped out, some water evaporates and some water solidifies. If the latent of water of vapourisation at 0°C is $2.1 \times 10^6 \text{ J/kg}$ and the latent heat of solidification of water is $3.5 \times 10^5 \text{ J/kg}$, calculate the maximum mass of ice that can be formed by this process.
3. The capacity of air conditioners is sometimes expressed in Watts or tons, the latter being the number of ice that can be frozen from water at 0°C in 24 hrs by the unit. Express the capacity of one-ton air conditioner in Watts.
4. A steel scale is to be prepared such that the milli-meter intervals are to be accurate within $5 \times 10^{-4} \text{ mm}$ at a certain temperature. Determine the maximum permissible temperature variation during the ruling of the milli-meter marks. $\alpha = 13.22 \times 10^{-6}/^\circ\text{C}$.
5. Clock A supports an iron pendulum and another clock B supports a brass pendulum. Both the clocks are accurate at 20°C . What will be the difference in their readings after one day if the temperature is 40°C ? $\alpha(\text{steel}) = 1.2 \times 10^{-5}/^\circ\text{C}$ and $\alpha(\text{brass}) = 2 \times 10^{-5}/^\circ\text{C}$.
6. Two walls I and II of the same thickness are made of heterogeneous metals, as shown in figure. In what case will the coefficient of thermal conductivity will be greater ? (For heat flow from left to right)

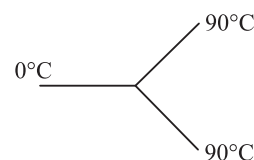


Choose the correct alternative. Only One choice is Correct.

7. Water of volume 2 litre in a container is heated with a coil of 1 kW at 27°C . The lid of the container is open and energy dissipates at rate of 160 J/s. In how much time temperature will rise from 27°C to 77°C ? [Given specific heat of water is 4.2 kJ/kg]

- (A) 7 min (B) 6 min 20 s (C) 8 min 20 s (D) 14 min

8. Three rods made of same material and having the same cross-section have been joined as shown in the figure. Each rod is of the same length. The left and right ends are kept at 0°C and 90°C respectively. The temperature of the junction of the three rods will be :



- (A) 45°C (B) 60° (C) 30°C (D) 20°C

9. In which of the following processes, convection does not take place primarily ?
 (A) sea and land breeze (B) boiling of water
 (C) heating air around a furnace (D) warming of glass of bulb due to filament
10. The coefficient of linear expansion of a crystal in one direction is α_1 and that in every direction perpendicular to it is α_2 . The coefficient of cubical expansion is :
 (A) $\alpha_1 + \alpha_2$ (B) $2\alpha_1 + \alpha_2$ (C) $\alpha_1 + 2\alpha_2$ (D) None of these

RADIATION

Section - 4

In radiation, heat is transferred from one body to other or to the surroundings even in the absence of any medium in the intervening space. Heat energy of the sun is transmitted to earth through radiations.

Radiation is only a mode of transfer of energy by transverse electromagnetic waves. While studying heat radiations (Radiant energy), we are concerned with thermal radiations which form the **infra-red region** of electromagnetic waves. All bodies emit heat to the surroundings at all temperatures and at all times. When the temperature of a body remains constant, it emits as much heat to the surroundings as it gains from them. The body is then in a state of **dynamic (thermal) equilibrium**.

Black Body :

A black body may be described as one that completely absorbs the radiations of all wavelengths falling upon it. There is no substance which exactly behaves like a black body as described.

Hollow Enclosure :

A hollow enclosure, kept at constant temperature behaves like a *black body*. If a small hole is made in such a enclosure, it will act as perfectly black body towards the incident radiations. Any ray getting into the enclosure through the hole will be internally reflected not be able to escape. Any radiation coming out of a narrow hole made in a *hollow enclosure* is generally referred to as **black body radiation** or **cavity radiation**.

Emissive Power :

It is the total amount of energy radiated by a body (at a particular temperature) per second per unit area of the surface.

E : energy radiated per second per unit area of the surface.

$$E = \frac{1}{A} \frac{\Delta Q}{\Delta t}$$

where A : surface area, $\frac{\Delta Q}{\Delta t}$: heat radiated per sec from the surface